Negative Queue Length: Queues for Quantum Information

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Entropy

- In classical Shannon's theory,

$$H(X) = -\sum_x P\{X = x\} \log P\{X = x\}.$$

The entropy H(X) measures the uncertainty of a data source, which is equivalent to how much information we can get when we learn the value of X .

- Given Y , the amount of information we gained from X can be measured by the conditional entropy:

$$H(X|Y) = H(X,Y) - H(Y).$$

always positive

Neumann's Entropy

- In the quantum world, the Neumann's entropy S (ρ) and conditional entropy S (ρ |σ) are defined by

$$S(\rho) = -tr(\rho \log \rho),$$

$$S(\rho|\sigma) = S(\rho, \sigma) - S(\sigma),$$

Mystery of Negative Quantum Entropy

- Quantum Theory is radically different with classical theory.
- Especially so-called entanglement of quantum states, or EPR states gives us negative quantum conditional entropy.

  → Long mystery...

Solution with Quantum Merging

- In 2005, Horodecki, Oppenheim and Andreas proposed the concept of partial information and quantum merging in [5].
- When S (ρ) < 0, they can store the excess of the certainty in the bank and use them in the future.
- Thus, the performance of quantum merging is certainly different with the classical information processing.

neurons (Fig. 1b). Furthermore, when the final stages of cell division are inhibited in hippocampal or cultured *Drosophila* neurons, the daughter cells contain two centrosomes. These cells form two long neurites that extend from positions directly overlying the two centrosomes (Fig. 1c). Although the experiments are carried out in different organisms, they indicate that centrosomes are required and sufficient for determining the position of axon outgrowth. Furthermore, they suggest the existence of a 'stage 0' in which cell polarity exists without any visible effect. This pre-existing polarity is used at later stages to direct neurite formation and axon specification.

These observations reveal exciting parallels between differentiating neurons and other cell types in which centrosomes initiate polarization. Shortly after fertilization, zygotes (single-celled embryos) of the nematode worm *Caenorhabditis elegans* become highly polarized along what will become the anterior–posterior axis[3]. This polarity is needed during the first cell division to segregate proteins differentially into what will become the two daughter cells, which will go on to have different fates. The axis of polarity in *C. elegans* zygotes is determined by the sperm entry-point, with polarization being initiated by an interaction between the centrosome (provided by the sperm) and the cell membrane[3].

The similarity between *C. elegans* and neurons extends to the molecular level. Both polarity processes seem to involve an evolutionarily conserved set of proteins known as Par proteins[3,4]. In *C. elegans*, Par-3 and Par-6 and the atypical protein kinase C (aPKC) localize to the anterior cell membrane, whereas Par-1 and Par-2 are concentrated posteriorly. In hippocampal neurons, Par-3 and Par-6 are found only in the axon, and if they are overexpressed, involved in cell signalling) and — like axon formation and *C. elegans* polarity — it requires Par-6 and aPKC (ref. 5). Although the function and distribution of these proteins during early neuronal differentiation (the hypothetical stage 0) are unknown, it is likely that they control the cross-talk between centrosomes and the cell membrane in neurons as well.

What happens downstream of the Par proteins? The protein Rac (another small GTPase) is primarily responsible for lamellipodium formation, and Par-3 interacts with the Rac activators Tiam1 and STEF (ref. 4). So Par-3 could be responsible for lamellipodium formation through localized Rac activation. Thus, by analogy to other cell-polarity events, we can already draw a molecular pathway for neurite outgrowth that can be tested in the hippocampal neuron culture model. It should be noted, however, that *Drosophila* axon outgrowth is independent of Par-6 and aPKC, and the proposed pathway must be verified experimentally before any further conclusions can be drawn.

The results of Calderon de Anda *et al.*[1] imply that the orientation of the final neuronal division is essential for correct wiring of the developing brain. Although such orientation is undoubtedly vital in invertebrates, its relevance in vertebrates is unclear[6]. The mechanisms responsible for the orientation of cell division have only recently begun to emerge in invertebrates[3]. Identification of those mechanisms in vertebrates will allow us to manipulate the orientation of cell division and test the effects on axon outgrowth — an experiment ultimately required to confirm the mechanism proposed by Calderon de Anda and colleagues. ■

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QUANTUM INFORMATION

Putting certainty in the bank

Patrick Hayden

A new way to manipulate quantum states resolves a long-standing conundrum about who knows what, and when and how, in the quantum world. The result is, as one has come to expect, startling and counterintuitive.

Claude Shannon's landmark 1948 theory of communication[1] tackles a nuts-and-bolts possible not just to be certain, but to be more than certain.

G-network

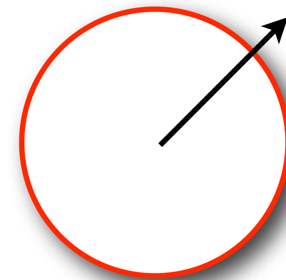
- In 1991, Gelenbe introduced the concept of negative customers in Markovian network setting[3].
- A negative customers are supposed to erase one positive customer in queue while the queue length is positive.
- The stability and product form solution are studied in the papers[3, 2, 11].

Negative customer cannot be stored...

Basics of Quantum Information Theory

- qubit: $|\psi\rangle = a|0\rangle + b|1\rangle,$

  $|0\rangle = (1,0)$ a $|1\rangle = (0,1)$



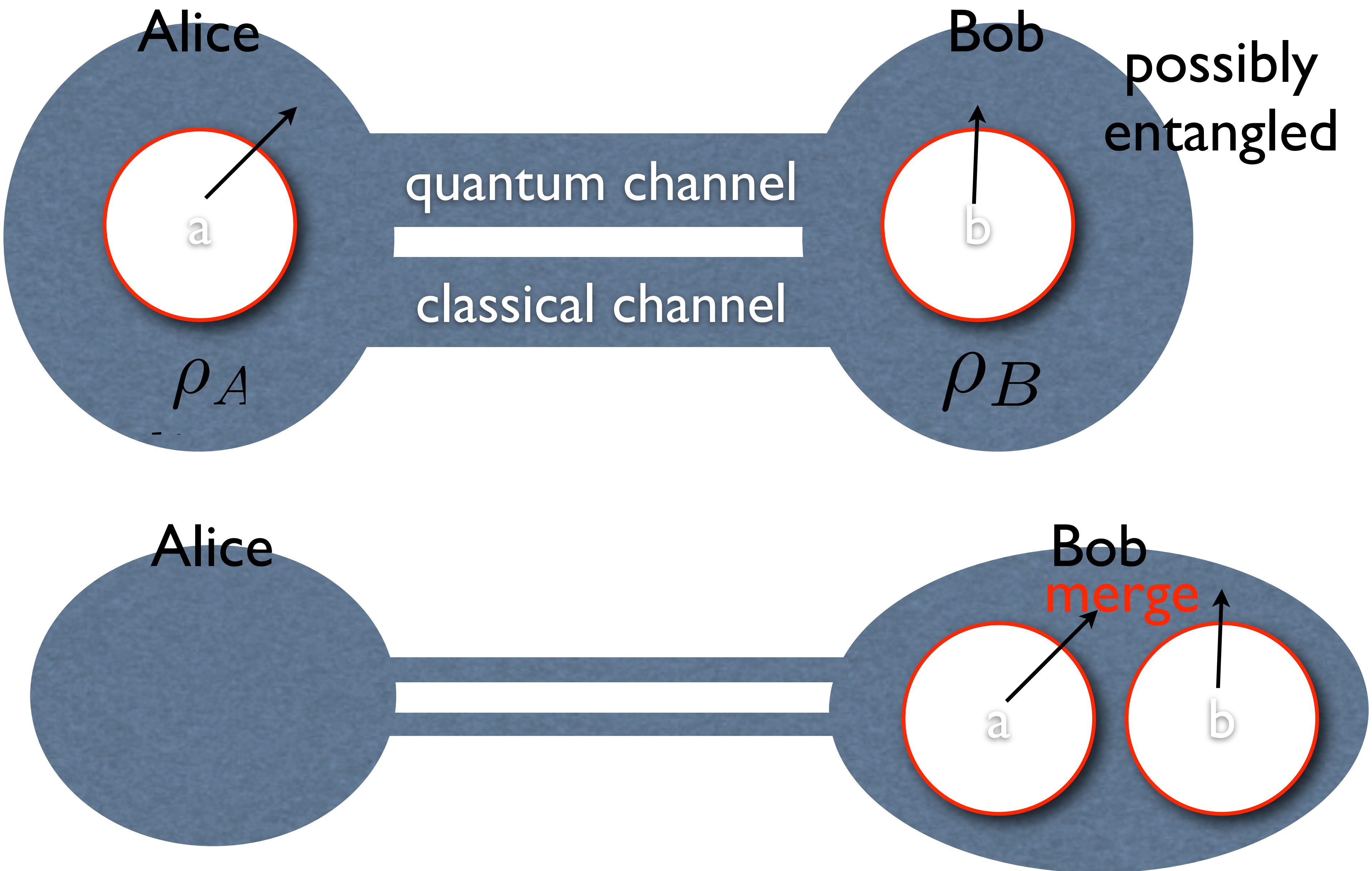
- EPR pair: $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle).$

  $\frac{1}{2}(|00\rangle\langle 00|_{AB} + |11\rangle\langle 11|_{AB})$

- Density operator $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|.$

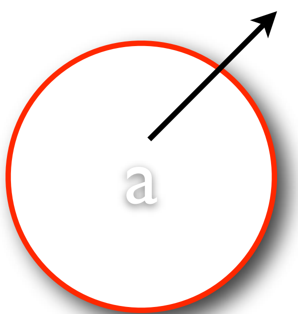
Example: $\rho = \frac{3}{4}|0\rangle\langle 0| + \frac{1}{4}|1\rangle\langle 1|.$

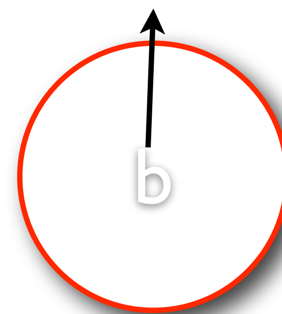
Quantum Merging



Case A: Independent pair

$$S(\rho_A|\rho_B) = 1$$

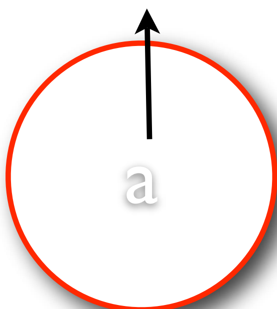




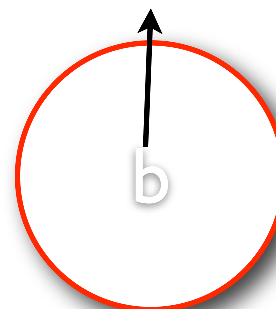
$$\rho_A = \frac{1}{2}(|0\rangle\langle 0|_A + |1\rangle\langle 1|_A), \qquad \rho_B = |0\rangle_B.$$

Send Alice's qubit to Bob, by using either quantum channel or quantum teleportation.

Case B: Classically strongly-correlated states



$$S(\rho_A|\rho_B) = 0$$

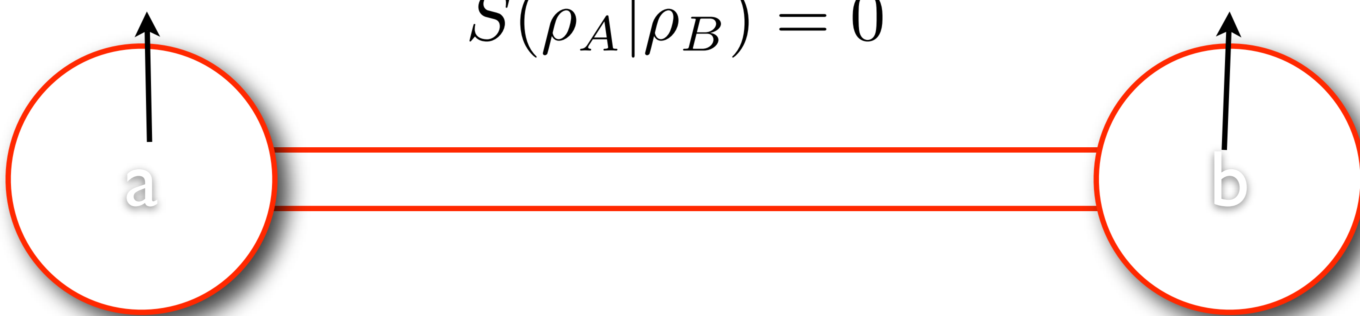


$$\rho_{AB} = \frac{1}{2}(|00\rangle\langle 00|_{AB} + |11\rangle\langle 11|_{AB}).$$

No need to send the information, Bob has to create one qubit synchronizing his qubit

Case C: Entangled state

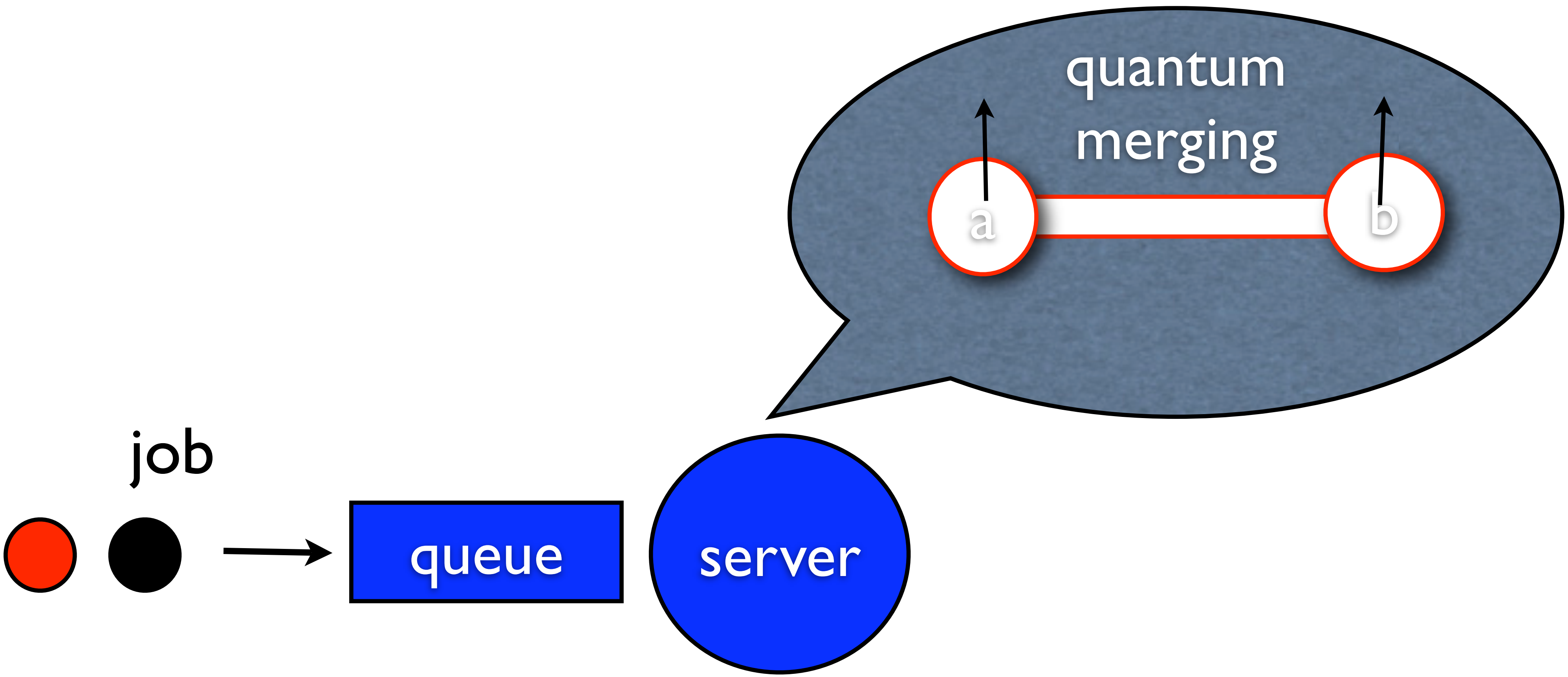
$$S(\rho_A|\rho_B) = 0$$



$$|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle).$$

EPR state is not only synchronized, but also has the ability to transfer other qubit, while Bob creates a new EPR pair locally.

Quantum Merging Server



Poisson arrival with rate λ

Positive and Negative customers

- Positive customer: qubit is independent with the receiver end.

  λ_p :arrival rate

  $G(x) = P\{S \leq x\}$:service time for quantum communication

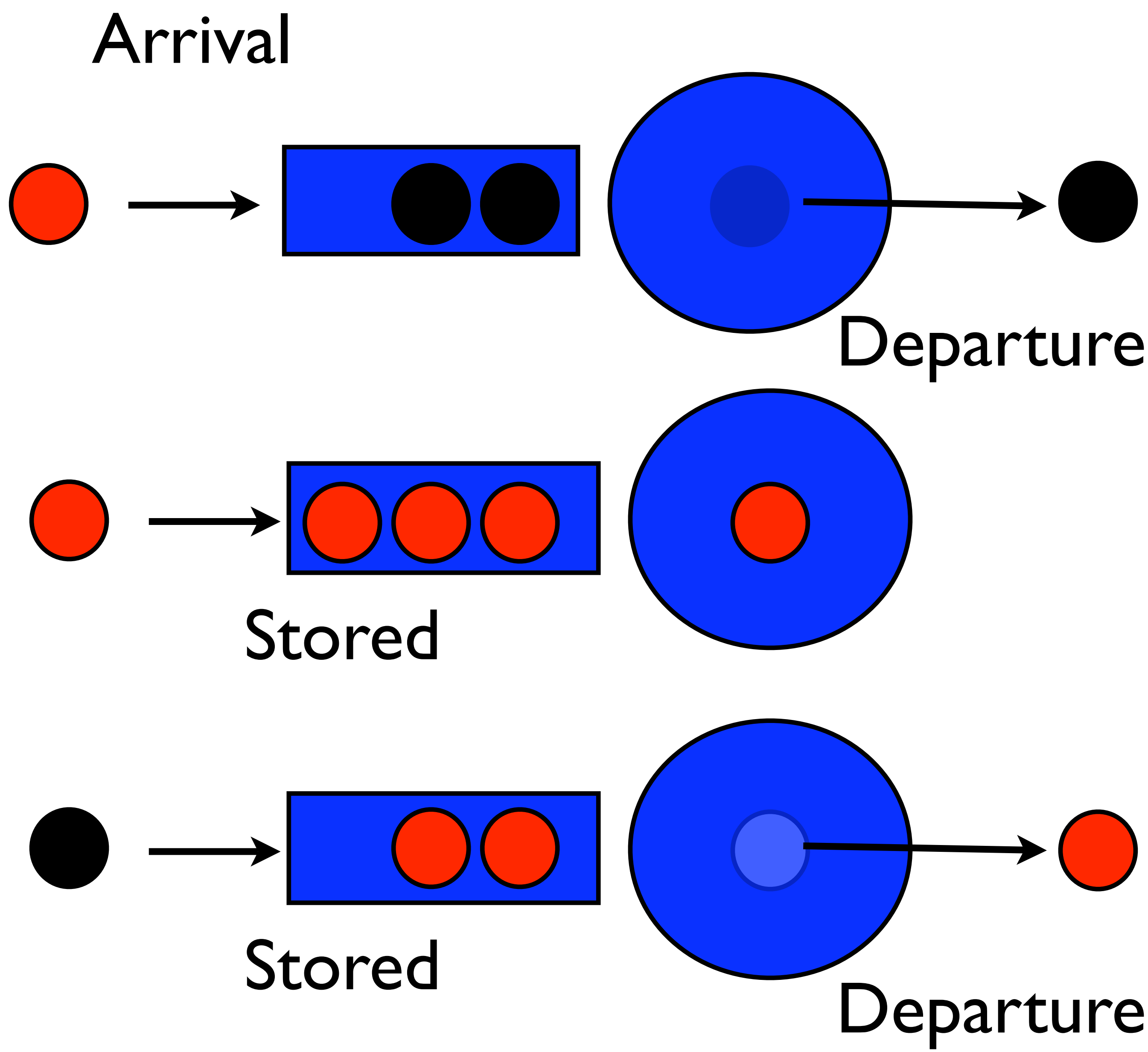
- Negative customer: qubit is entangled with the qubit at the receiver end.

  $'\lambda_n$:arrival rate

Negative Customers

- Whenever there is a positive customer waiting, those EPR pair of the negative customers will be used to merge the positive customer's qubit immediately.
- When there is no positive customer in the system, we can save EPR pairs for future use.

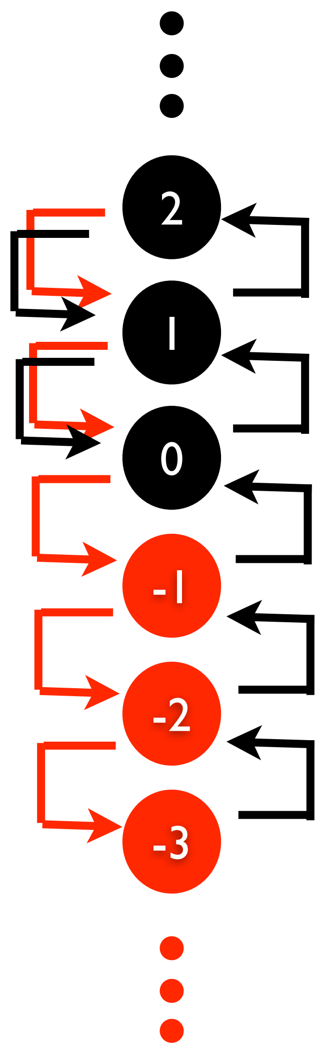
M/G/1± queue



State transitions

λp: arrival of positive customer

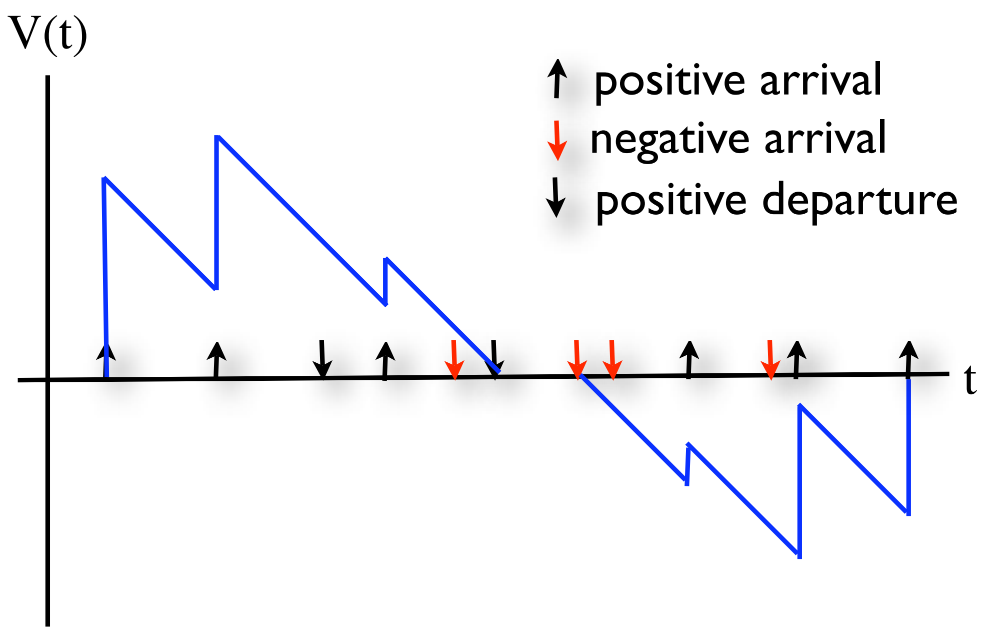
μ: departure of positive customer



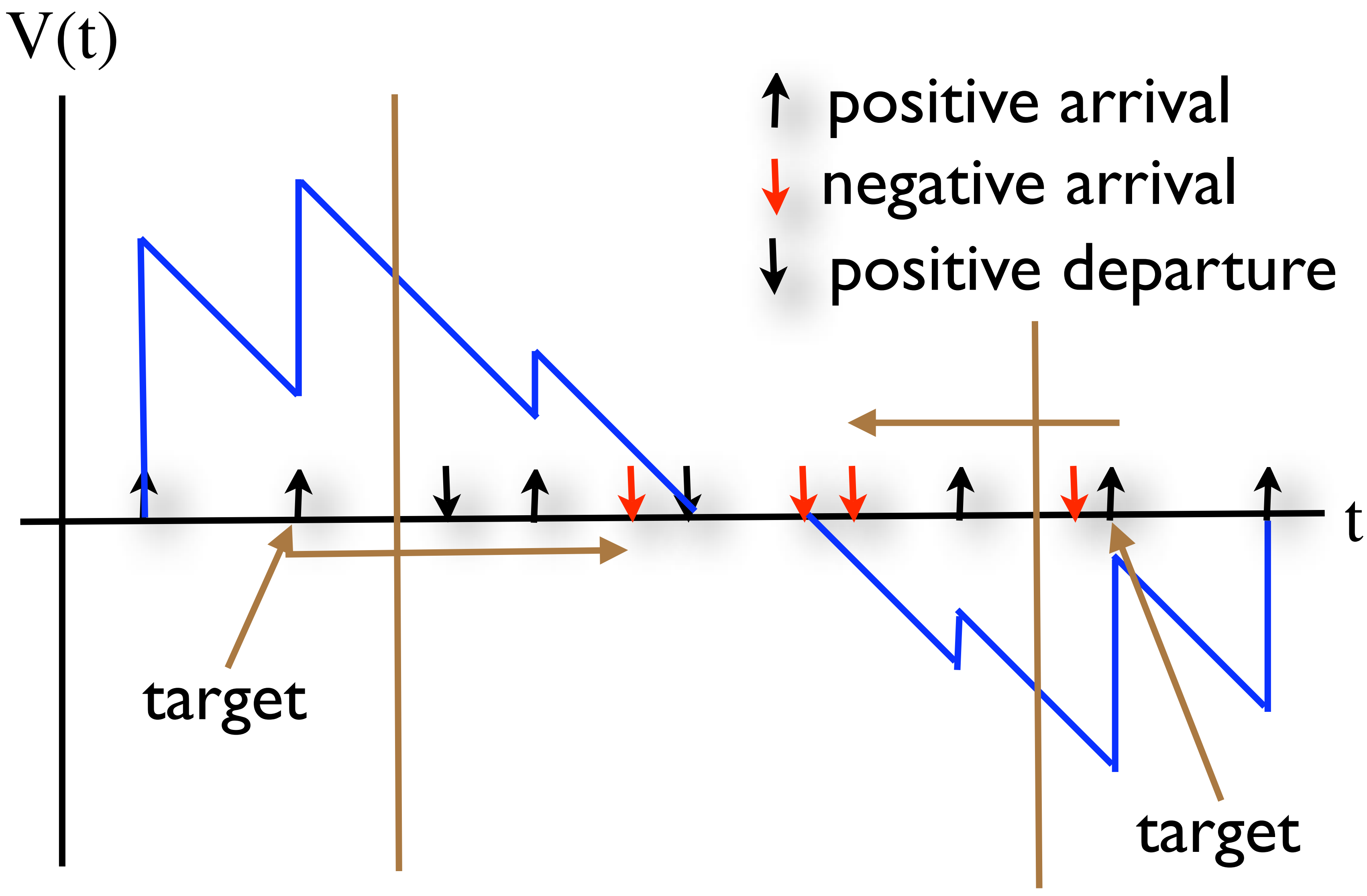
λp: arrival of positive customer

The virtual waiting time

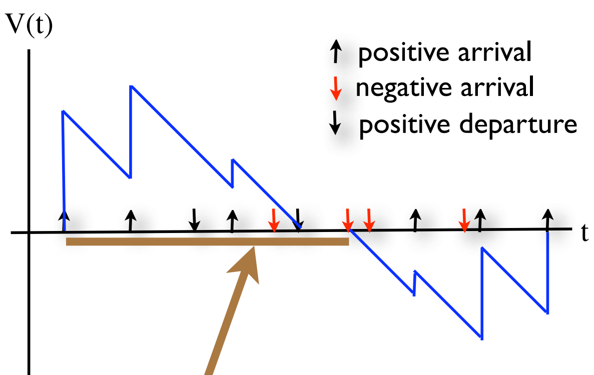
Suppose an additional virtual positive customer has been arrived at our system at the time t. We allow this customer to arrive prior to the time t. The virtual waiting time V(t) is the difference between t and the time when this virtual customer starts his service.



Why Negative Virtual Waiting Time?

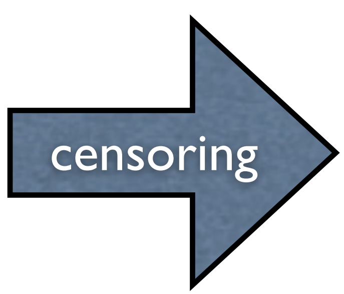


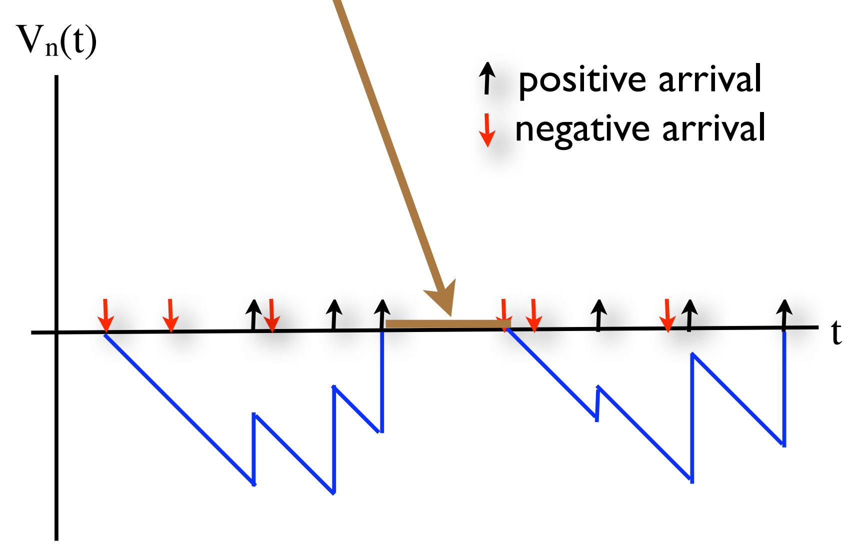
Censoring and Idle-time Modification



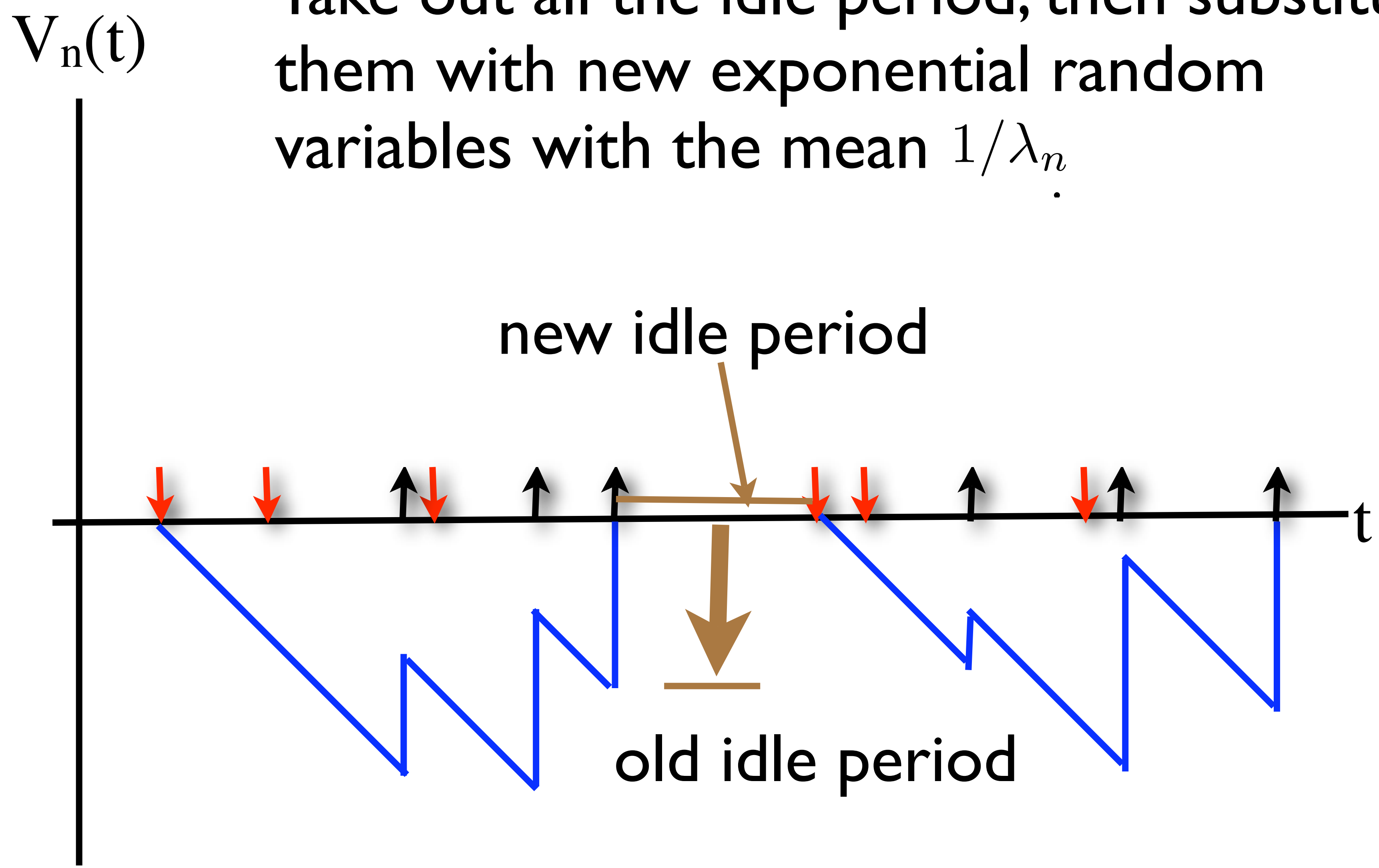
Note that idle periods are exponentially distributed with the mean $1/(\lambda_p + \lambda_n)$

Cycle started by a positive customers are censored





Take out all the idle period, then substitute them with new exponential random variables with the mean $1/\lambda_n$.



The resulted modified process is an M/M/1 queue with the arrival rate λn and the service rate λp

Mean Virtual Waiting Time for Negatively Censored Process

$$E[W_n(t)] = \frac{\lambda_n/\lambda_p}{\lambda_p - \lambda_n}.$$

$$\begin{aligned} E[-V_n(t)] &= \frac{E[Y_n] + 1/\lambda_n}{E[Y_n] + 1/(\lambda_p + \lambda_n)} E[W_n(t)] \\ &= \frac{\lambda_n + \lambda_p}{2\lambda_p(\lambda_p - \lambda_n)}. \end{aligned}$$

Positively-Censored Process.

- We can use the similar argument for the positively-censored process.
- This time the queue can be analyzed as an M/G/1 queue.
- Adjusting the replacement of the idle time, we have

$$E[V_p(t)] = \frac{E[Y_p] + 1/\lambda_p}{E[Y_p] + 1/(\lambda_p + \lambda_n)} E[W_p]$$
$$= \frac{(\lambda_p + \lambda_n)E[S_p^2]/2}{(1 + \lambda_n/\mu_p)(1 + \lambda_p/\mu_p)}.$$

E[V(t)]

positive cycle | negative cycle

$$E[V(t)] = \frac{\mu_p^2(\lambda_p - \lambda_n)E[S_p^2]/2}{(\mu_p - \lambda_n)(\mu_p - \lambda_p)} - \frac{\lambda_n(\mu_p - \lambda_p)}{\lambda_p(\mu_p - \lambda_n)(\lambda_p - \lambda_n)}.$$

$$S_p = min(S, T_n),$$

$$\mu_p = 1/E[S_p]$$

Mean Actual Waiting Time

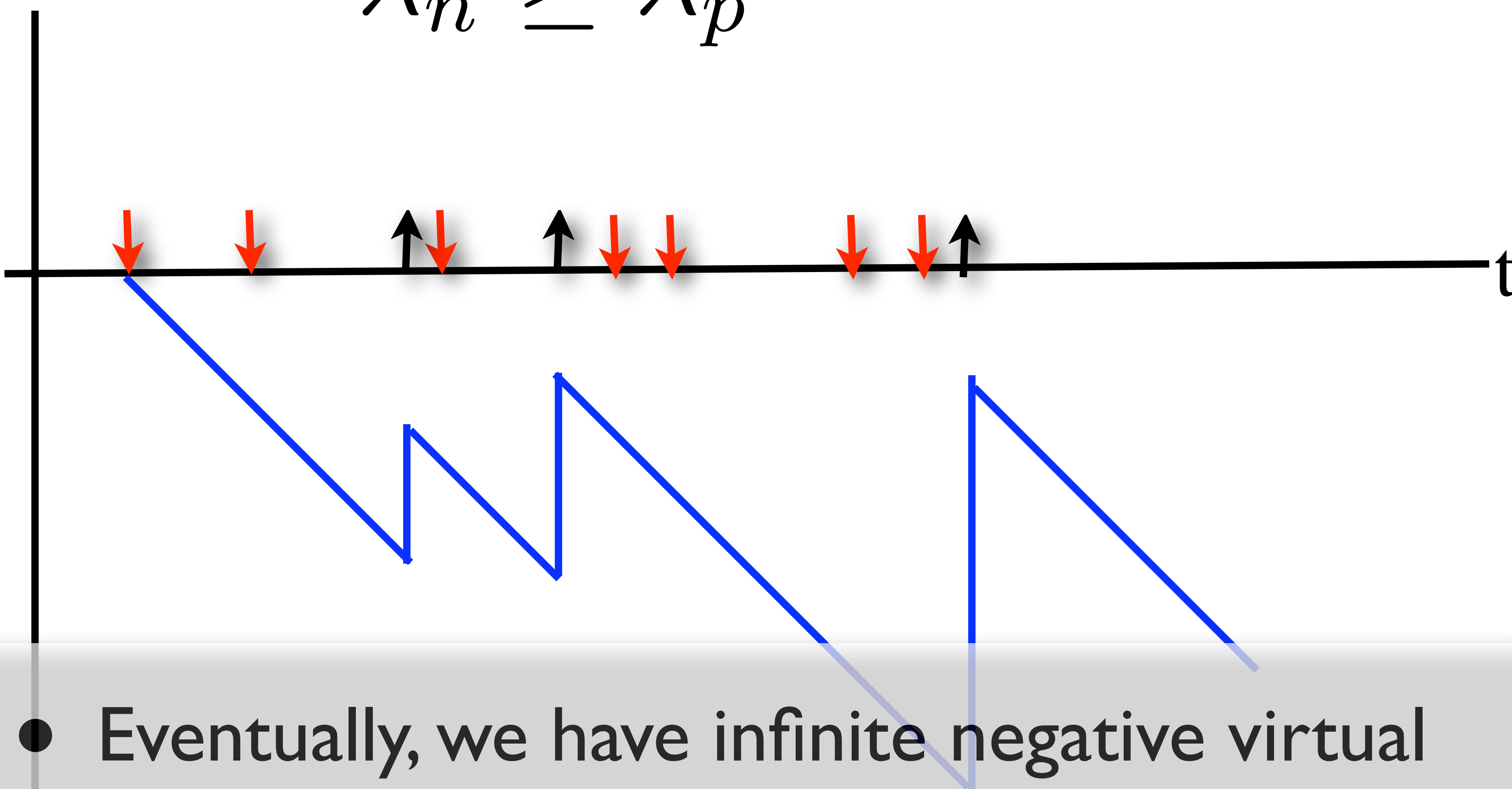
$$E[W] = \frac{\lambda_p \mu_p^2 (\lambda_p - \lambda_n) E[S_p^2]/2}{(\mu_p - \lambda_n)(\mu_p - \lambda_p)(\lambda_p + \lambda_n)},$$

Stability condition

$$\lambda_n < \lambda_p < \mu_p.$$

Large Negative Customer Rate

$$\lambda_n \geq \lambda_p$$



- Eventually, we have infinite negative virtual waiting time. Thus, no actual waiting time!

Large Positive Arrival Rate

- Even the arrival rate of positive customers is larger than service rate,

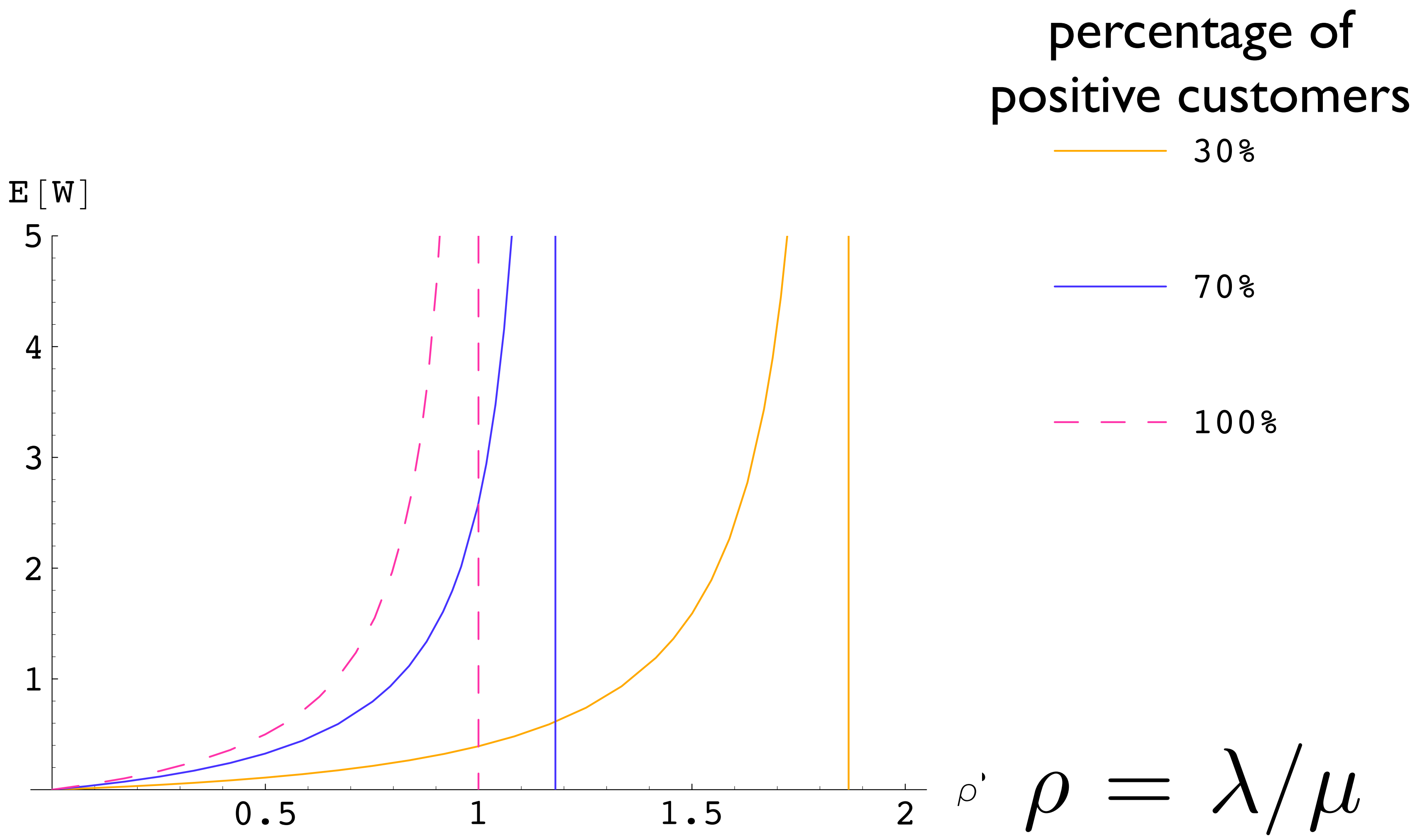
$$\mu < \lambda_p < \mu_p = 1/E[\min(S, T_n)]$$

still, we have finite mean waiting time.

$$E[W] = \frac{\lambda_p \mu_p^2 (\lambda_p - \lambda_n) E[S_p^2]/2}{(\mu_p - \lambda_n)(\mu_p - \lambda_p)(\lambda_p + \lambda_n)},$$

However, if $\lambda_p > \mu_p$, then the waiting time will blow up.

Quantum Merging Server
M/D/1± queue



$$\rho = \lambda/\mu$$

Positive Customer and Mean Waiting Time